Continued fractions and module structure of extensions of p-adic fields.

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Classical Galois module theory has its starting point in the normal basis theorem: every Galois extension posseses a normal basis: a basis consisting in the Galois orbit of some element in the top field. Rings of integers of extensions of local or global fields are not so well behaved, as they need not admit a normal integral basis in general. In fact, Noether [Noe32] showed that a Galois extension of local fields posseses a normal integral basis if and only if it is tamely ramified. However, when the ring of integers is free over an object depending on the Galois group, its associated order, one can construct an integral basis similar to a normal one. This approach was introduced by Leopoldt [Leo59] and it motivated subsequent research on the Galois module structure of wildly ramified extensions. Among these works, F. Bertrandias, J.P. Bertrandias and M.J. Ferton [BF72; BBF72] solved completely the problem for cyclic degree p extensions of p-adic fields, finding criteria for the freeness over the associated order in terms of the continued fraction expansion of the ramification number of the extension over the prime p. However, the proof of this surprising connection is barely sketched and has remained mysterious for decades. In this talk, we review the techniques leading to the proof of their result and present an adaptation to degree p extensions with dihedral normal closure, for which we use the setting provided by Hopf-Galois theory.

References

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